

# Relation Between a Three Parameter Formula for Isotope Shifts and Staggering Parameters

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## Abstract

It is noted that the staggering parameters used to describe even-odd effects for isotope shifts can in some cases exhibit very rapidly varying behavior as a function of neutron number. On the other hand a three parameter formula (3P) with fixed coefficients can explain the same behavior.

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Table I: The Staggering Parameters Using the 3P Formula and A,B,C from Blaum

n	$\gamma_n$			
	Ar	K	Ca	Ti
0	0.388	0.660	0.052	-0.387
2	-0.086	0.460	-1.844	-128.000
4	-3.805	-4.000	3.844	2.418
6	2.908	1.853	1.948	1.705

A three parameter formula was employed by Zamick [1] and by Talmi [2] to describe the isotope shifts in a single j shell  $r^2(n) = nC + n(n-1)/2A + [n/2]B$  where  $[n/2]=n/2$  for even n and  $(n-1)/2$  for odd n. Zamick argued that this should have the same form as a binding energy formula described in DeShalit and Talmi [3] because both  $r^2$  and the two-body interaction are rotational scalars. We can use this formula to obtain the staggering parameter which is designed to emphasize the even-odd behavior of isotope shifts.

The staggering parameter  $\gamma$  is defined by

$$\gamma_n = 2[r^2(n+1) - r^2(n)]/[r^2(n+2) - r^2(n)] \quad (1)$$

One can easily show the relation between  $\gamma_n$  and the 3P formula

$$\gamma_n = 1/(1 + M) \quad \text{where} \quad M = y/(1 + nx) \quad (2)$$

where  $y=(A+B)/2C$  and  $x=A/C$

We thus see that there are only two independant constants for calculating staggering parameter. That there are only two

is clear from the fact that multiplying A,B and C by the same constant will not change the staggering parameter although

it will of course change the isotope shift.

By examining Eq. (2) we can quickly get the following results:

- a. If  $A+B=0$  then  $\gamma_n = 1$  for all n (no staggering).
- b. If  $A=0$  there is no n dependence to the staggering.
- c. For very large  $nA$  the staggering parameter approaches one. (Since A is usually very small it is not clear if this limit will be reached in practice.)

Table II: Staggering Parameters for Ar and K Isotopes from Experiment

n	$\gamma_n$	
	Ar	K
0	0.527	0.274
2	-0.337	-1.385
4	-3.037	-0.174
6	1.461*	1.278

d. Treating  $n$  as a continuous variable we can, for small  $n$  perform a Taylor's expansion up to terms linear in  $n$  and get

$$\gamma_n = \gamma_0[1 + \frac{An}{C}y/(1+y)]$$

where  $\gamma_0 = 1/(1+y)$ .

Since  $A$  is usually negative the curve starts on a downslope (e.g. from Blaum [8] we have for argon  $A = -0.018(1)$ ,  $B = 0.119(13)$ ,  $C = 0.032(5)$ ).

In this work we consider the isotopes of Ar, K, Ca and Ti. Although the 3P formula was derived for a single  $j$  shell, Wolfart et. al. [4] showed that it worked if this condition is relaxed. We utilize their findings plus the fact that the 3P formula has been applied by the following experimental groups, Andle et al. for Ca[5], Martensson-Pendrill et. al. for K[6], Gansky et. al. for Ti[7] and Blaum et. al. for Ar[8]. We follow most closely the work of Blaum et. al. [8] where the parameters  $A$ ,  $B$ ,  $C$  for all these nuclei have been compiled. From their work we have calculated the staggering parameters shown in Table 1.

Alternatively, one can get the  $\gamma_n$ 's for Ar isotopes without the 3P formula. Rather we get the values of  $r^2(n)$  directly from Table 3 of Blaum [8]. However they have no value of  $r^2$  for  $A=45$  so we substitute the 3P prediction. For K isotopes we use the results of A-M Martensson-Pendrill et. al. [5].

\* Since there is no measurement for  $45\text{Ar}$  we use the 3P formula for this nucleus.

From the definition of the staggering parameters we see that getting a value close to plus or minus infinity is not so mysterious. One gets a magnitude of infinity if the  $A+2$  nucleus has the same radius as the  $A$  nucleus. The large negative dips followed by return to positive, that is displayed in all the above nuclei is easily obtained with the 3P formula.

Let us discuss the behavior of  $\gamma$  vs  $n$  for the Argon isotopes. In a work in which the

B(E2)'s and magnetic moments of the even-even isotopes of these nuclei were calculated by Robinson et. al. [9] it was noted that the experimental B(E2)'s rose steadily from A=38 to 42 (midshell for the neutrons) but then steadily decreased for A=44 and 46. The respective values in units of  $e^2 fm^4$  were 130(10), 330(40), 430(100), 345(41) and 196(39). This could explain why  $\gamma$  becomes large and negative for n=4 (which involves 43,44,45 Argon).

The linear term in the 3P formula causes  $r^2$  to increase with n. The fact that  $^{44}Ar$  has a smaller B(E2) than  $^{42}Ar$  would by itself cause a decrease of  $r^2$  for  $^{44}Ar$  relative to  $^{42}Ar$ . If these two effects were to cancel completely,  $\gamma$  would have a magnitude of infinity.

In some sense this shows a deficiency in the sheer definition of  $\gamma$ . It was designed to emphasize odd-even effects but sometimes the even-even difference in the denominator can swamp the odd-even difference in the numerator.

The advantage of the using the staggering parameter is that because it is defined as a ratio some atomic physics parameters that are not well determined get factored out. In particular there is the F factor which related the observed frequencies in atomic transitions to isotope shifts. Thus some experimentalists can measure staggering parameters without

being able to measure isotope shifts. Early papers which obtain staggering parameters include those of Kuhn et. al. [10]

on tellurium and H.H. Sroka et. al. on mercury isomers [11]. They do not discuss the 3P formula.

The 3P formula has also been applied to heavier nuclei. For example, Talmi [2] analyzed the data on lead isotopes by Thompson et. al. [12] with this formula. These same isotopes were addressed by W.H. King et al. [13] and Anselment et al.

[14]. These last two authors show that the parameters used in ref [2] show a very flat curve of the staggering parameter

versus neutron number for the lead isotopes in disagreement with experiment. This is especially shown in a figure

in ref [14] where it is shown that this parameter decreases rapidly with decreasing neutron number. One can see,

based on the comments in this work why this is the case. The parameters used in ref [2] were

$A = -0.001$ ,  $B = 0.050$ ,  $C = 0.058$ . Note that the chosen A is extremely small. As mentioned before for A=0 the staggering

parameter is independent of  $n$ .

The values of  $G_{AM}$  found by King [13] for Isotopes 207, 205, 203, 201 and 199 are respectively

0.75, 0.51, 0.45, 0.39, and 0.12 (corresponding to  $n = 0, 2, 4, 6$  and 8). If we fit the first and last of these we obtain

$y = .3333 x = A/C = -.1193$ . In ref [2] the value of  $x$  is -0.0172. With the new set of parameters the value of  $G_{AM}$

are 0.75, 0.695, 0.61, 0.46, and 0.12. The slope with decreasing  $n$  is much better than in ref [2] but is far from perfect.

The problem is due to the fact that we are dealing with more than one shell and more than one value of the ground state spin.

M. R. Pearson et. al. [15] used the 3P formula to analyze the bismuth isotopes as well as lead. R. A. Sheline [16] considered cesium and barium isotopes. The latter authors discussed anomalous staggering for which the odd  $A$  isotope has an unusually large radius. They attributed this to octupole deformation which is more prevalent for odd  $A$  as compared to even  $A$ . We will not discuss these nuclei here but we use them to indicate how widespread the use of the 3P formula is.

One reason the 3P formula is in fairly wide use by experimentalists is that more fundamental approaches run into difficulties. Early on Uhrer and Sorensen [17] were able, with their pairing plus quadrupole model, to obtain good results for the even-even to even-even mass isotope shifts but in their words “the odd-even staggering effects observed in the isotope shifts cannot be obtained”. Sagawa et. al. [18] attempted to explain the staggering in lead isotopes by couplings to giant monopole and quadrupole resonances. They found that monopole couplings were much more important but the staggering came mainly from the quadrupole couplings. They have a nice formula relating the staggering in isotope shifts to the staggering of  $B(E2)$ ’s but unfortunately the staggering is much too small. In the work of Blaum [8] on argon isotopes it is noted that spherical Hartree Fock leads to isotope shifts that are too small. A simulation of deformed H-F increase these shifts in better agreement with experiment but neither of these approaches gives odd-even staggering. They imply that an HFB approach might lead to success [8]. But their main analysis is with the 3P formula.

We have an added comment on a very recent development concerning  $B(E2)$ ’s in  $^{46}\text{Ar}$ . In ref [9] we calculate this and compare with values of Scheit et al [19]. In units of  $e^2\text{fm}^4$

Robinson et al.[9] obtained 535 as compared with Scheit's value of 196.

A large value was also obtained in the shell model calculations in the Scheit paper. But a paper just appeared by

Mengoni et al. [20] where the  $B(E2)$  was claimed to be much larger than the previous measurements. They obtained a value

of  $570 \text{ e}^2 \cdot \text{fm}^4$ . This is significantly larger than that for  $^{44}\text{Ar}$   $335 \text{ e}^2 \cdot \text{fm}^4$  (for  $^{44}\text{Ar}$  good agreement with theory is obtained[9])

While it is initially gratifying that the new experiment agrees with the shell model calculations some serious

questions arise when we try to connect with isotope shifts. In ref [8] one sees that the charge radius of  $^{46}\text{Ar}$

is smaller than that of  $^{44}\text{Ar}$ . One would expect that if  $^{46}\text{Ar}$  has a bigger deformation than  $^{44}\text{Ar}$  it should have

a larger charge radius. It is perhaps premature to draw any definitive conclusions but it will be interesting to see

how this all plays out.

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